# Multi-Robot Control for Surveillance of Dynamic Population Distributions 

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#### Abstract

This paper presents a multi-robot control system that enables the robots to monitor dynamic populations of individuals. In this work, the population's state is modeled as a geo-referenced graph, where individuals occupy nodes on the graph and use edges to travel between nodes. At any given time some fraction of the population may occupy each node. The control system presented uses calculated transition probabilities to dictate robot motion about the same graph. The system has the following properties: 1) robot navigation is collision-free, 2) robots only require local information when calculating their transition probabilities, and 3) the entire system dynamics are provably stable in that over time the distribution of robots at nodes will equal the distribution of individuals occupying the nodes. Simulation results indicate that for a 100 node graph occupied by 60 robots, the average difference between the steady state robot node distribution and desired node distribution was less than 0.01 . Field test results with real quadcopters demonstrate that the controller can be implemented on a real system.


Index Terms-Multi-robot system, Controller, Distribution Tracking, UAV Surveillance, Shark Tracking

## I. Introduction

Persistent environmental monitoring is a common need across a variety of applications. Examples include patrolling an area to detect intruders, surveying a location for air quality data, or monitoring animals in their natural habitat. Such tasks can oftentimes be dangerous, mundane, and/or costly for human beings, and are thus well suited for automation with robots [1]. Accordingly, a recent field of research has emerged around the use of multi-robot systems for environmental monitoring [2].
This paper presents an algorithmic solution to the control of a multi-robot system for monitoring a spatial population distribution that is abstracted as a graph, where each node represents a geo-referenced location where individuals of the population being monitored may be located. The individuals can move between nodes across edges of the graph, and the distribution of individuals across nodes is dynamic. Proposed is a decentralized collision-free stochastic controller that coordinates multiple robots to provably match the expected robot's node visit likelihood with the population's distribution of individuals across the nodes.


Fig. 1. Two depictions of graph abstraction for the multi-robot controller

To motivate this work, it is being applied to a fish monitoring application where nodes of a graph represent potential locations of fish. This model motivates the goal of developing a stochastic controller for multi-robot distributions. An example is illustrated in Fig. 1(a), where a graph is anchored to a particular area off the California coast, in which fish are known to aggregate. This graph can also be used as a navigation map for quadcopters equipped with downward facing cameras. The robots can fly between nodes and land on an Autonomous Surface Vehicle (ASV), which is stationed at the central node 0 , for recharging. A similar stochastic framework is leveraged here, and the contributions of this work include:

- A decentralized, collision-free, stochastic control system that matches a node's robot visitation likelihood with the proportion of the tracked population residing at the node.
- Hardware experiments performed with real quadcopters demonstrating system feasibility.
This work differs from prior work in that the controller that can track dynamic distributions. It also allows higher priority nodes, (those with higher fractions of the distribution being tracked,) to be visited at a frequency proportional to their priority level. The distributions can be updated dynamically and the controller can respond in real-time. Combined with the benefits of being decentralized and producing collision-free navigation, the proposed controller provides a novel approach to a general multi-robot coordination problem.


## II. Control System

## A. Problem Definition

The problem can be described as coordinating $n$ quadcopters in the set $Q=\left\{q_{0}, \ldots, q_{n-1}\right\}$ that are restricted to navigate along edges, and between nodes of a graph $G=(V, E) . V$ is the set of nodes and $E$ is the set of edges. The state of a quadcopter $x_{t}\left(q_{i}\right)$ at discrete time $t$ is associated with some node $v_{j}$, such that $x_{t}\left(q_{i}\right)=v_{j}$. More generally, the state of all quadcopters is defined as $X_{t}(Q)=\left[x_{t}\left(q_{0}\right) \ldots x_{t}\left(q_{n-1}\right)\right]$ where $X_{t}(Q) \in V$. The set of nodes that are occupied by quadcopters is denoted as $V_{o c c}$
To further define the problem, the likelihood that a robot occupies any particular node $v_{j} \in V$ at time $t$ is $p_{j, t}$, where $\sum_{j} p_{j, t}=1$ for all $t$. Each node $v_{j}$ also has a time varying fraction $p_{j, d e s, t}$ of the population of individuals being tracked by the quadcopters such that $\sum_{j} p_{j, d e s, t}=1$ for all $t$. The goal of the controller is determine robot motion such that $p_{j, \text { des }, t}=p_{j, t}$ for all $t$.

Also navigating the nodes of $G$ are $m$ ASVs in the set $A=\left\{a_{0}, \ldots, a_{m-1}\right\}$. At periodic intervals, quadcopters must visit a node occupied by an ASV to land and recharge. That is, there must exist some time step $t_{r}$ within the recharge period $T_{r}$ where each quadcopter $q_{i}$ is collocated with an ASV $a_{l}$ at some vertex $v_{k}$. Specifically $x_{t_{r}}\left(q_{i}\right)=x_{t_{r}}\left(a_{l}\right)=v_{k}$. For work done in the remainder of this paper, each ASV will be stationary and reside at a single node.

If at every time step $t$, any quadcopter $q_{i}$ chooses an action $u_{i, t}: x_{t-1}\left(q_{i}\right) \rightarrow x_{t}\left(q_{i}\right)$, and the action set for all quadcopters at time $t$ is $U_{t}=\left\{u_{0, t} . . u_{n, t}\right\}$, then the problem statement can be formulated as follows:

$$
\begin{array}{cl}
\underset{U_{t}}{\operatorname{minimize}} & \sum_{t=0}^{t_{\max }} \sum_{j=0}^{|G|} p_{j, t, d e s}-p_{j, t}\left(X_{t}(Q), U_{t}\right) \\
\text { subject to } & X_{t}(Q) \in V \forall t \\
& \exists t_{r} \in T_{r} \mid x_{t_{r}}\left(q_{i}\right)=x_{t_{r}}\left(a_{l}\right) \forall i \\
& \left(x_{t-1}\left(q_{i}\right) \rightarrow x_{t}\left(q_{i}\right)\right) \in E \forall t, \forall i
\end{array}
$$

The constraints state that 1) all quadcopters must reside on nodes during the discrete time steps, 2) there exists some time step within the recharge period at which each quadcopter is on an ASV, and 3) quadcopters must move only along edges.

## B. Graph Construction

For this study, the nodes in $V$ are generated by clustering geo-referenced positions from a historical data of fish population activity [3]. Methods such as k-means clustering were used to extract ideal locations for nodes. Other graph extraction algorithms can also be used.

Once the nodes are generated, directed edges are added to connect all nodes in $V$ sequentially to create a uni-directional cyclic tour. Various TSP methods can be used to generate this tour. Since the tour is uni-directional, collision negotiations only occur between adjacent quadcopters, making the collision checking local and decentralized.

## C. Motion Controller

To determine how quadcopters navigate along the tour, a stochastic motion controller that requires minimal run time and communication requirements is proposed. This controller specifies the transition probability $\tau_{j+1, j, t}$ at every time step $t$ such that in real time operations, any quadcopter $q_{i}$ located at $v_{j}$ will randomly select to either stay at $v_{j}$ or navigate to the next edge connected node $v_{j+1}$ with likelihood $\tau_{j+1, j, t}$. Within this context, the control action $u_{i, t}$ uses a uniform sampling function $\operatorname{rand}() \in[0,1]$ to determine the next node to visit:
$u_{i, t}= \begin{cases}x_{t-1}\left(q_{i}\right) \rightarrow x_{t}\left(q_{i}\right)=v_{j+1} & \text { if } \operatorname{rand}()<\tau_{j+1, j, t} \\ x_{t-1}\left(q_{i}\right) \rightarrow x_{t}\left(q_{i}\right)=v_{j} & \text { else }\end{cases}$
These transition probabilities can be encapsulated in a transition matrix $\mathbf{R}_{t}$, where the $j$ th row and $k$ th column element in $\mathbf{R}_{t}$ is denoted as $\tau_{j, k, t}$. Then, given the current distribution of quadcopters $\mathbf{p}_{t}=\left[p_{0, t} \ldots p_{|V|-1, t}\right]^{T}$, the quadcopter distribution update can then be described by:

$$
\begin{equation*}
\mathbf{p}_{t+1}=\mathbf{R}_{t} \mathbf{p}_{t} \tag{2}
\end{equation*}
$$

Thus, the goal is to set the elements $\tau_{j, k, t}$ of $\mathbf{R}_{t}$ such that $\mathbf{p}_{t}$ converges to the desired distribution $\mathbf{p}_{d e s, t}=$ $\left[p_{0, \text { des }, t} \ldots p_{|V|-1, \text { des }, t}\right]^{T}$ over time. The following values for the elements of $\mathbf{R}_{t}$ are proposed:

$$
\begin{align*}
\tau_{j, j, t} & = \begin{cases}1 & \text { if } v_{j+1} \in V_{o c c} \\
\tau_{\text {stay- }} & \text { else if } v_{j-1} \in V_{o c c} \\
\tau_{\text {stay }+} & \text { else }\end{cases} \\
\tau_{j+1, j, t} & = \begin{cases}0 & \text { if } v_{j+1} \in V_{o c c} \\
\tau_{\text {move- }} & \text { else if } v_{j-1} \in V_{o c c} \\
\tau_{\text {move }+} & \text { else }\end{cases} \tag{3}
\end{align*}
$$

In Eq. (3), $\tau_{j, j, t}$ determines the likelihood of staying at node $v_{j}$, while $\tau_{j+1, j, t}$ determines the likelihood of moving from $v_{j}$ to the next node of a tour $v_{j+1}$. Since a quadcopter must either stay or move on, then $\tau_{j, j, t}+\tau_{j+1, j, t}=1$.

To eliminate the possibility of collisions, both the likelihood of staying at current node $v_{j}$ is set to 1 , and the likelihood of transitioning to the next node $v_{j+1}$ is set to 0 , if $v_{j+1}$ is occupied, (i.e. $v_{j+1} \in V_{o c c}$ ).

Equations (4)-(7) provide the transition probabilities for cases where the next node on a tour is not occupied and can be transitioned to. These transition probabilities are functions of individual node errors $e_{j, t}=p_{d e s, j, t}-p_{j, t}$ and constant proportional control gains $K_{a}$ and $K_{b}$.

$$
\begin{gather*}
\tau_{s t a y-}=\frac{\left[1-p_{j+1, t}+\frac{K_{a} e_{j, t}}{p_{j, t}}+\frac{K_{b}\left(e_{j-1, t}-e_{j+1, t}\right)}{p_{j-1, t}\left(p_{j, t}\right)}\right]}{1-p_{j+1, t}}  \tag{4}\\
\tau_{\text {stay }+}=\frac{\left[1-p_{(j+1), t}+\frac{K_{a} e_{j, t}}{p_{j, t}}\right]}{1-p_{(j+1), t}} \tag{5}
\end{gather*}
$$

$$
\begin{gather*}
\tau_{\text {move- }}=\frac{-\left(\frac{K_{a} e_{j, t}}{p_{j, t}}+\frac{K_{b}\left(e_{j-1, t}-e_{j+1, t}\right)}{p_{j-1, t}\left(p_{j, t}\right)}\right)}{1-p_{j+1, t}}  \tag{6}\\
\tau_{\text {move }+}=\frac{-K_{a} e_{j, t}}{p_{j, t}\left(1-p_{j+1, t}\right)} \tag{7}
\end{gather*}
$$

As shown in in Eq. (3), if the next node is not occupied, the controller next considers whether or not the previous node $v_{j-1}$ is occupied, (i.e. $v_{j-1} \in V_{o c c}$ ). If it is, the controller uses proportional control on node density errors for nodes $v_{j-1}, v_{j}, v_{j+1}$ to determine the likelihood of staying versus moving on, (Eq. (4) and (6)). This ensures that quadcopters at the previous node $v_{j-1}$ don't get stuck at that node when there is less there to observe, (e.g. $p_{j-1, \text { des }}$ is low).

If the previous node is not occupied, a proportional controller using error associated with the current node $v_{j}$ is used to set the transition probability, (Eq. (5) and (7)).

Ideally, the individual node errors will all decay to zero over time, such that for all nodes the likelihood of robots visiting a node matches the fraction of individuals being monitored visiting that node. It can be proved, (outside the scope of this abstract), that setting the control gains $K_{a}$ and $K_{c}$ appropriately will lead to stable error dynamics that decay errors to zero over time.

A few notes on the transition probability calculations. First, quadcopters only need to know information about previous, current, and next nodes: $e_{j-1, t}, e_{j, t}, e_{j+1, t}$, allowing the system to be decentralized. Second, they ensure collision-free navigation by setting likelihoods of transitioning to occupied nodes to 0 . Third, the desired distribution $\mathbf{p}_{\text {des }, t}$ can be estimated and initialized with prior knowledge (i.e. known population migratory behavior) or possibly estimated in realtime as it changes, thereby enabling the tracking of dynamic distributions. Fourth, the calculations in equations (3)-(7) require relatively insignificant computational power and can be accomplished in real-time. Finally, the following theorem can be proven, although outside the scope of this abstract:
Theorem II.1. The error dynamics for $\boldsymbol{e}_{t}$ will be stable for gain values of $0<K_{a}<1$ and $0<K_{b}<\frac{1}{2}$.

## III. Simulation Tests

All simulation tests were written in MATLAB. To test the controller, simulations were run on graphs with random desired tracking distributions. The number of nodes in the graph were varied between values of $5,10,20,40,60,80$, and 100. The number of quadcopters was set to values of $1,10,20,30,40,50$, and 60 percent of the nodes. For each node/quadcopter number combination, 100 trials were conducted and performance was evaluated as the average error from Eq. (3) across all nodes, across all 100 trials.

As shown in Fig. 2 the error values predominantly falls within $<0.05$. The error plot indicates that the controller converges to the desired distribution with some steady state error. Error increases as the percentage of nodes occupied increase and simultaneously the number of nodes decrease.


Fig. 2. Average absolute error as a function of number of nodes and robots.


Fig. 3. Trajectories of hardware trials conducted at Linde Field at Harvey Mudd College.

## IV. Field Tests

Physical experiments were conducted with three modified 3DR Solo quadcopters running OpenSolo. Flight times range from 10 to 20 minutes. The flight controller of the 3DR Solos were upgraded from a Pixhawk 2.1 Black Cube to a Pixhawk 2.1 Green Cube flashed with Arducopter v3.5. Autonomous functionality was programmed with Dronekit. The script was run on a remote computer that connected to each quadcopter through a designated WiFi link. For these preliminary tests, the system controller was centralized for ease of implementation.

Fig. 3 depicts the trajectories logged during some of the hardware trials. As shown in the figures, theses trials validated the ability for the controller to coordinate the collision-free motion of multiple quadcopters around the graph.

## REFERENCES

[1] M. Dunbabin and L. Marques, "Robots for environmental monitoring: Significant advancements and applications," IEEE Robotics Automation Magazine, vol. 19, no. 1, pp. 24-39, March 2012.
[2] D. Portugal and R. Rocha, "A survey on multi-robot patrolling algorithms," in Technological Innovation for Sustainability, L. M. CamarinhaMatos, Ed. Berlin, Heidelberg: Springer Berlin Heidelberg, 2011, pp. 139-146.
[3] K. D. Smith, S. Hsiung, C. White, C. G. Lowe, and C. M. Clark, "Stochastic modeling and control for tracking the periodic movement of marine animals via auvs," in 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Oct 2016, pp. 3101-3107.

